By the end of our Level 2 Course the students should be able to solve the following problems.

Level 2. Sample Problems

1. Given that $x + y = 3$ and $xy = 2$. Find the value of $x^4 y + xy^4$.

2. There is a pile of 2019 stones on a table. You are allowed to perform the following operation: you choose one of the piles containing more than 1 stone, throw away one stone from that pile and divide it into two smaller (not necessarily equal) piles. Is it possible to reach a situation in which all the piles on the table contain exactly 7 stones?

3. Three prime numbers are such that their product is 103 times greater than their sum. Find all such numbers.

4. Show that no integer a satisfies $a^2 + 3a + 5 \equiv 0 \pmod{121}$.

5. Let x and y be positive real numbers. Show that
$$\frac{x}{x^4 + y^2} + \frac{y}{y^4 + x^2} \le \frac{1}{xy}$$

6. One corner square was cut off from a 64×64 square of graph paper. Prove that the obtained figure can be dissected into L-shape trominos consisting of three squares.

7. Point M is chosen inside the triangle ABC, such that $\angle BMC = 90° + \frac{1}{2}\angle BAC$. Line AM contains the circumcenter of the triangle BMC. Show that the lines BM and CM contain the circumcenters of the triangles AMC and AMB respectively.

8. 9999 squares of size 2×2 are cut off from the board 299×299. Is it always possible to cut off one more 2×2 square?

9. Is it possible that the number of the form $5^n + 1$ be divisible by the number of the form $5^k - 1$?

10. Given an acute triangle ABC and let its altitudes AD, BE, CF be its altitudes and H be its orthocenter. Let H_A, H_B, H_C be the midpoints of the segments AH, BH, CH and A_1, B_1, C_1 be the midpoints of the sides BC, AC, AB respectively. Prove that the lines $H_A A_1$, $H_B B_1$, $H_C C_1$ are concurrent.