



42 Points | Advanced Math Training Program

By the end of our Level 3 Course the students should be able to solve the following problems.

Level 3. Sample Problems

1. Find all natural k and n , such that $k^5 + 5n^4 = 81k$.
2. Point K is chosen on the side BC of the triangle ABC . KN is the angle bisector of the angle AKC . Lines BN and AK intersect at F . Lines CF and AB intersect at D . Find $\angle DKN$.
3. Show that for all natural k the number $k^2 + k + 1$ is not divisible by 101.
4. Find all nonnegative real numbers $a_1 \leq a_2 \leq \dots \leq a_n$ satisfying

$$\sum_{i=1}^n a_i = 96, \quad \sum_{i=1}^n a_i^2 = 144, \quad \sum_{i=1}^n a_i^3 = 216$$

5. Let DM be the diameter of the incircle of a triangle ABC where D is the point at which the incircle touches the side AC . The extension of BM meets AC at K . Prove that $AK = CD$.
6. Given two non-intersecting circles ω_1 and ω_2 . Prove that the midpoints of the four common tangent lines to ω_1 and ω_2 lie on the same line.
7. Prove that

$$\sqrt{2 + \sqrt[3]{3 + \dots + \sqrt[1993]{1993}}} < 2$$

8. Prove that it is not possible to divide any set of 18 consecutive positive integers into two disjoint sets A and B , such that the product of the elements in A equals the product of the elements in B .
9. In the triangle ABC , $AB > AC$. An external bisector of the angle $\angle BAC$ intersects the circumcircle of the triangle ABC at E . Let F be the foot of perpendicular from E to line AB . Prove that $2AF = AB - AC$.
10. Find all integer solutions of the equation

$$\frac{a^7 - 1}{a - 1} = b^5 - 1$$